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HEMANT KUMAR, MAHMOOD AHMAD PATHAN, AND HARISH SRIVASTAVA

ABSTRACT. In the present paper, we consider an anomalous diffusion problem in two dimensional space involving Caputo time and Riesz-Feller fractional derivatives and then solve it by using a series involving bilateral eigen-functions. Also, we obtain a numerical approximation formula of this problem and discuss some of its particular cases.

1. Introduction

By a set of axiom, definitions and methods of fractional calculus many processes in the nature are modelled (see Kilbas et al. [8], Miller and Ross [12], Samko et al. [15] and Podlubny [14]). One of these processes is an anomalous diffusion which is a phenomenon that occurs in complex and non-homogeneous mediums.

The anomalous diffusion may be based on generalized diffusion equation which contains fractional order space and/or time derivatives (see Mainardi et al. [9]). Metzler and Klafter [11] and Turski et al. [18] presented the occurrence of the anomalous diffusion from the physical point of view and also explained the effects of fractional derivatives in space and/or time to diffusion propagation. Agrawal [1, 2] applied an analytical technique by using eigen-functions for a fractional diffusion-wave system.

Mathai, Saxena and Haubold [7, 10] investigated the solution of a unified fractional reaction diffusion equation associated with Caputo derivative as the time-derivative and Riesz-Feller fractional derivative (see Ciesielski et al. [3]) as the space-derivative. They have derived its solution by the application of the Laplace and Fourier transforms in a compact and closed form in terms of the H-function.

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