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MULTIDIMENSIONAL LAGUERRE TRANSFORMS

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ABSTRACT : In the present paper, we introduce multidimensional Laguerre transform to present its certain interesting applications to the theory of generalized multiple hypergeometric functions of several variables including multivariable *H*-function of Srivastava and Panda [11, 12]. The various operational formulas thus obtained are believed to be new, these results may be useful in deriving new and known properties of special functions involved. In the last section, we shall also introduce another multidimensional Laguerre transform to derive interesting results.

1. INTRODUCTION

Chandel [1] introduced multidimensional Laplacian oprator to give integral representations of Lauricella's multiple hypergeometric functions of several variables [9]. Chandel [2] further used this operator to give integral representations of multiple hypergeometric functions ${}^{(k)}_{(1)}E^{(n)}_{D}$ and ${}^{(k)}_{(2)}E^{(n)}_{D}$ of Exton [7, 8]. Further Chandel and Dwivedi [3, 4] introduced multidimensional Whittaker transforms of Lauricella's multiple hypergeometric functions [9], Exton [7, 8] and generalized multiple hypergeometric function of Srivastava and Daoust [10] (also see Srivastava and Manocha [14, p. 64 (18), (19), (20)]). Recently Chandel and Kumar [5] have made applications of above multidimensional integral transforms to derive the results involving Srivastava and Panda's *H*-function of several complex variables [11, 12].

In the present paper, making an appeal to the result due to Erdélyi et. al. [6, p. 292]

$$\int_0^\infty Z^{\rho} e^{-z} L_n^{(\alpha)}(z)dz = \frac{(-1)^n\Gamma(\rho-\alpha+1)\Gamma(\rho+1)}{n!\Gamma(\rho-\alpha-n+1)}, \quad Re\ (\rho) > -1$$

We introduce multidimensional Laguerre transform defined by

$$(1.1)\quad L^{(\alpha,\beta,m)}_{\gamma_1 \dots \gamma_n}\{\} = \frac{(-1)^m m!\,\Gamma(\beta-\alpha-m+\gamma_1+\dots+\gamma_n)\Gamma(\gamma_1+\dots+\gamma_n)}{\Gamma(\beta-\alpha+\gamma_1+\dots+\gamma_n)\Gamma(\beta+\gamma_1+\dots+\gamma_n)\Gamma(\gamma_1)\dots\Gamma(\gamma_n)}$$