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On Unified Advection-Dispersion Problem and its Fourier Series Solution Involving Volterra Integral Equation

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ABSTRACT

In the present investigation, we introduce an unified space-time fractional advection-dispersion equation involving Caputo time fractional derivative of order β $(\beta > 0)$, Riesz-Feller space fractional derivatives of order γ $(0<\gamma<1)$ and asymmetry θ_1 $(|\theta_1| \le \min(\gamma, 1-\gamma))$ and of order α $(1<\alpha\le 2)$ and asymmetry θ_2 $(|\theta_2| \le \min(\alpha, 2-\alpha))$. Then, we consider a Fourier series to obtain its solution involving Volterra integral equation. We also evaluate its numerical approximation formula and discuss some of its particular cases.

1. Introduction

Regarding the linearity of the differential operators Kontecky [17] and Matsuda and Ayabe [22] studied the series solution of semi-differential equations (see also Oldham and Spanier [24, p.159]).

King et al. [15, p.123] described the Fourier series solution of ordinary one-dimensional diffusion equation for temperature distribution in the bar.

Özdemir et al. [25] obtained an analytic solution of fractional diffusion equation by applying Fourier series and also evaluated its numerical approximation formula.

Gorenflo, Luchko and Zabreiko [20] have solved the Cauchy problem and represented its series solution involving Mittag-Leffler function $E_{\alpha,\beta}(.)$ defined by

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)} \quad (z,\beta\in\mathbb{C};\Re(\alpha)>0) \tag{1.1}$$

where $\mathbb{C}$ is a set of complex numbers and $\Gamma(.)$ is the Gamma function (see Erdélyi et al. [5] and Kilbas et al. [14]).

Many researchers such as Kilbas et al. [14], Oldham et al. [24], Podlubny [26], Samko et al. [27], and Mathai, Saxena and Haubold [19] presented a systematic study with analytical properties and applications of fractional derivatives, integrals and differential equations. Recently, Diethelm [3] has developed the theory and analysis of fractional differential equations involving Caputo type differential operators. Our work is concerning with the method developed by Diethelm [3] in the spaces of integrable, absolutely continuous and orthogonal functions.

Let $\Omega=[a,b], (-\infty\le a<b\le\infty)$ be a finite or infinite interval of the real axis $\mathbb{R}=(-\infty,\infty)$, $L_p(a,b)(1\le p\le\infty)$ is the set of those Lebesgue complex-valued functions f on Ω for which $\|f\|_p<\infty$ where

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