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ON A BI DIMENSIONAL BASIS INVOLVING SPECIAL FUNCTIONS FOR PARTIAL IN SPACE AND TIME FRACTIONAL WAVE MECHANICAL PROBLEMS AND APPROXIMATION

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Abstract

In this work, we construct a fractional time dependent wave mechanical problem consisting partial in space and time fractional derivatives and solved it on introducing a bi dimensional basis function involving Hermite and Mittag – Leffler functions. Then, we use it to approximate the solution of above two variable wave mechanical problem and then discuss its various cases in which it is computable.

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1. Introduction

The Hermite functions play an important role in the wave mechanical treatment of the harmonic oscillator. (see Mott and Sneddon [11, p. 50], Eravanis [1]). It also has great importance in the application to the quantum theory of radiation [19]. In this connection, the time independent Schrödinger equation corresponding to harmonic oscillator of point mass m with vibrational frequency ν is given by (Sneddon [17])

$$\frac{d^2\Psi_n}{dx^2}+\left(\frac{2T}{h\nu}-x^2\right)\Psi_n=0, \tag{1}$$

where, T is the total energy of the oscillator and h is the Plank's constant. Here, the wave functions Ψ_n have the property that

$$\Psi_n \to 0 \text{ as } |x| \to \infty; \tag{2}$$

and

$$\int_{-\infty}^{\infty}|\Psi_n|^2dx = 2\pi\sqrt{\frac{m\nu}{h}} \tag{3}$$

In Eqn. (1), there is the Hermite function $\Psi_n = e^{-\frac{x^2}{2}}H_n(x)$ and $\frac{2T}{h\nu} = 2n+1$, where, $H_n(x)$ being the Hermite polynomials [13] for all $n = 0,1,2,\ldots$, satisfying the differential equation

$$\frac{d^2H_n(x)}{dx^2} - 2x\frac{d}{dx}H_n(x) + 2nH_n(x) = 0 \tag{4}$$

The $H_n(x)$ is defined by

$$H_n(x) = \begin{cases} (-1)^n e^{x^2}\frac{d^n}{dx^n}e^{x^{-2}}, (\textit{the operational formula}) \\ (2x)^n\sum_{r=0}^{\infty}\frac{\Gamma\left(-\frac{n}{2}+r\right)\Gamma\left(\frac{1}{2}-\frac{n}{2}+r\right)}{\Gamma\left(-\frac{n}{2}\right)\Gamma\left(\frac{1}{2}-\frac{n}{2}\right)\Gamma(1+r)}(-x^{-2})^r, (\textit{the series formula}) \end{cases} \tag{5}$$

The function Ψ_n also satisfies the recursion formula

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