Fractional Reduced Differential Transform Method To Analytical Solution Of Fractional Order Biological Population Model

HarishSrivastava*1 and Dharmendra Badal*2 Kunal Srivastava*3

*Department of Mathematics, Asst Professor:D.V Postgraduate College Orai, Jalaun(U.P) **Department of Mathematical Sciences &Computer Applications, Asst Professor:Bundelkhand University Jhansi(B.U Campus), (U.P)., India

***Department of Sciences for the subject of mathematics, Research Scholar : P.K University Karera Shivpuri, (M.P)., India

Abstract - In this paper, we obtain analytical solution to the non-linear fractional order biological population model by using fractional reduced differential transform method. We presented some examples are provided to check the effectiveness, accuracy and performance of proposed work. The results and figure show that the proposed method is very convenient.

2010 Mathematics Subject Classification: 35S10, 33E12.

Keywords:, Biological population model, Caputo derivative, Initial value problem, Mittag-Leffler function, Reduced differential transform method.

1. INTRODUCTION

Nowadays there is increasing attention paid to fractional order differential equations and their broad applications in mathematics, physics and engineering [2],[3],[11],[12],[13],[17] such as anomalous transport disordered system, earthquake modeling the diffusion problem, the phenomena in electromagnetic acoustic viscoelasticity etc. have been widely spread in the recent years. In the present paper, fractional reduced differential transform method is considered. This method for the fractional order differential equations provides the analytical solutions for both the linear and non-linear fractional order differential equations in the form of power series. The method was presented by Keskin and Oturanc [6], Srivastava V.K.et al. [16] and they applied the fractional reduced differential transform method to non-linear fractional order differential equations.

We extend the fractional reduced differential transform method to time fractional-order biological population model. The representive generalized time fractional-ordered nonlinear biological population diffusion equation is given as

$$D_{t}^{\alpha}u(x,y,t) = \left(u^{2}(x,y,t)\right)_{xx} + \left(u^{2}(x,y,t)\right)_{yy} + f\left(u(x,y,t)\right)$$
(1.1)

with the initial condition

$$u(x, y, 0) = f(x, y),$$
 (1.2)

where $t > 0, x, y \in R$ (set of real numbers) and u(x, y, t) denotes the population density and f(u) represents the population supply due to births and deaths. Also, $f(u) = hu^a (1 - ru^b)$ with h, a, r, b are real numbers. If we take special value of numbers then